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The scattering of an electromagnetic wave by a free electron

D G Ashworth and R C Jennison
University of Kent at Canterbury, Canterbury, CT2 7NK, UK

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Abstract. The laws of conservation of energy and momentum are used to eliminate Planck's constant from the usual relationship for Compton scattering. The resulting expression is identical to the relativistic Doppler equation and it is shown that, in the proper frame of the interaction, Snell's law is obeyed and the scattering is specular. This leads to the possibility that the reflection occurs in a region rotating about the centre of the electron at almost the velocity of light.

The interaction of a photon with a free electron, as depicted in figure 1(*a*), was first described by Compton (1923). The photon, of initial energy $E_\gamma = h\nu$, is deflected by an angle ϕ and has a lower energy $E'_\gamma = h\nu'$ after collision. The electron, at rest prior to the collision with energy $E_0 = m_0c^2$, moves off after the collision with velocity V and energy $E = mc^2$ at an angle ψ with respect to the initial direction of propagation of the photon. The symbols ν and ν' refer to the frequency of the photon before and after collision. m_0 is the rest mass of the electron and $m = m_0(1-V^2/c^2)^{-1/2}$ is the mass of the electron when it is travelling at velocity V. c is the velocity of electromagnetic waves in free space and h is Planck's constant.

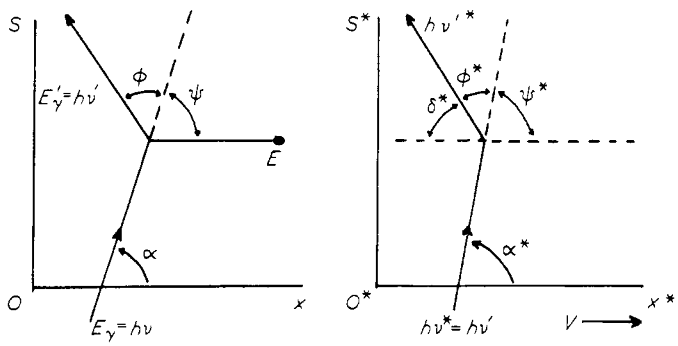

Figure 1. The Compton effect; (*a*) in *S*, (*b*) in *S**.

The law of conservation of energy allows us to write

$$h\nu + m_0c^2 = h\nu' + mc^2 \tag{1}$$

and the law of conservation of momentum gives

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + mV\cos\psi \tag{2}$$

and

$$0 = \frac{h\nu'}{c}\sin\phi - mV\sin\psi. \tag{3}$$

Equations (1)–(3) combine to give the familiar 'Compton equation'

$$\cos\phi = 1 - \frac{m_0 c^2}{h}\left(\frac{1}{\nu'} - \frac{1}{\nu}\right). \tag{4}$$

If, instead of the above procedure, we combine equations (1) and (2) we find that

$$\frac{h\nu'}{c}(1-\cos\phi) = c(m_0 - m) + mV\cos\psi$$

in which we can substitute for $\cos\phi$ from equation (4) to give

$$\frac{\nu'}{\nu} = \frac{1-(V/c)\cos\psi}{(1-V^2/c^2)^{1/2}}. \tag{5}$$

This procedure has led to a complete elimination of Planck's constant which would seem to imply that perhaps the concept of 'light quanta' is unnecessary for an understanding of certain aspects of the Compton effect. Let us now extend this idea even further by invoking the concepts of the relativistic Doppler effect and the aberration formulae. Consider two reference frames: the laboratory frame S (figure 1(a) is drawn in S) and the frame S^* which is moving with velocity V, with respect to S, in the direction of the electron after collision. The x axis of S is drawn parallel to the x^* axis of S^* such that the direction of motion of the electron is along $0x$. Let the frequency of light in S be ν and the frequency of the same light in S^* be ν^*. If the direction of the light in S is at an angle α to $0x$ and at an angle α^* to 0^*x^* in S^* then, see figure 1, by the relativistic aberration equation and Doppler equation (see for example Ditchburn 1952), we have that

$$\cos\alpha^* = \frac{\cos\alpha - V/c}{1-(V/c)\cos\alpha} \tag{6}$$

and

$$\nu^* = \frac{\nu[1-(V/c)\cos\alpha]}{(1-V^2/c^2)^{1/2}}. \tag{7}$$

Applying equation (7) to the situation depicted in figure 1(a), we have that $\alpha \equiv \psi$ and hence,

$$\nu^* = \frac{\nu[1-(V/c)\cos\psi]}{(1-V^2/c^2)^{1/2}}$$

and, by equation (5), $\nu^* = \nu'$. The frequency of the photon before collision, as seen from the frame of reference in which the electron is at rest after the collision, is therefore the same as the frequency of the photon after collision in the laboratory frame. The Compton effect in S and S^* is shown in figure 1.

From equation (6)

$$\cos(\phi^* + \psi^*) = \frac{\cos(\phi+\psi) - V/c}{1-(V/c)\cos(\phi+\psi)} \tag{8}$$

but, $\cos(\phi+\psi) = \cos\phi\cos\psi - \sin\phi\sin\psi$, therefore, using equations (2) and (3),

$$\cos(\phi+\psi) = \frac{\nu}{\nu'}\cos\psi - \frac{mVc}{h\nu'}. \tag{9}$$

Using equations (1)–(3) it can readily be shown that

$$h\nu = \frac{m_0(m-m_0)c^3}{mV\cos\psi - c(m-m_0)}$$

and

$$h\nu' = \frac{m(m-m_0)c^2(c-V\cos\psi)}{mV\cos\psi - c(m-m_0)},$$

which, in conjunction with equation (9), give

$$\cos(\phi+\psi) = \frac{-(\cos\psi - V/c)}{1-(V/c)\cos\psi}\{\equiv -\cos\psi^*\} \tag{10}$$

therefore, from (8), $\cos(\phi^*+\psi^*) = -\cos\psi$, hence, from figure 1(*b*), $\delta^* = \psi$. Also, from (7),

$$\nu'^* = \frac{\nu'[1-(V/c)\cos(\phi+\psi)]}{(1-V^2/c^2)^{1/2}}$$

which, by equation (10), gives

$$\nu'^* = \frac{\nu'(1-V^2/c^2)^{1/2}}{[1-(V/c)\cos\psi]},$$

or, by (5), $\nu'^* = \nu$.

If we define the 'proper' frame, S_p, of the interaction as the one in which the electron is always at rest, with 0_px_p drawn parallel to $0x$ and 0^*x^* as in figure 2, then the frequency of the light is the same after collision as before collision, ie ν. Also, the angle of incidence is equal to the angle of reflection, both angles being measured from the x_p axis, ie Snell's law is obeyed in the 'proper' frame of the event. Thus, Compton scattering is an example of pure specular reflection in the 'proper' frame of the scattering element.

In the laboratory frame the effect is similar to simple scattering from a particle of small inertia which recoils against the momentum of the impinging electromagnetic wave. The scattered light is modified in frequency as though emitted from a receding source and is therefore Doppler shifted to the red.

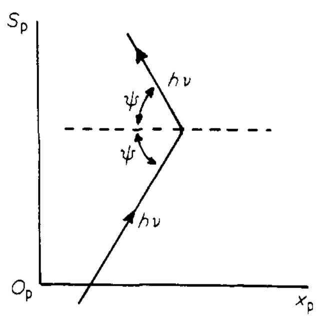

Figure 2. The Compton effect in the 'proper' frame of the electron.

We note the recent work by Lamb and Scully (1969) in which they provide an explanation of the photo-electric effect without using the concept of photons but state that the Compton effect is *entirely* a photon phenomenon. This is in keeping with the traditional view given in the following statement by Born (1957): ... 'Compton... found that the radiation scattered at an angle of less than 90° possesses a greater wavelength than the primary radiation, so that the ν' of the scattered wave, contrary to the prediction of the classical theory, is smaller than the ν of the incident radiation. On the principles of the wave theory, this phenomenon is unintelligible.' It would appear, however, that this aspect of the Compton effect is fully explicable on classical grounds.

An extension of the classical model leads to the conclusion that the specular nature of the reflection may indicate that in the proper frame of the interactions the scales of length and time are such that the incoming wave does not appear to engulf the reflector. The indeterminacy of the angle of recoil could then result from the spinning of the reflecting surface. Thus if the reflection does not occur in a region at rest relative to the centre of the electron but in a region or at a nodal surface (Jennison 1973) which is rotating at a very high velocity relative to the centre, the radiation at that surface will appear shifted considerably to the blue (higher energy) end of the spectrum. The time scale of the interaction will be reduced and the wavelength of the radiation in the frame of the node could be sufficiently short for specular conditions to prevail without the necessity for the incoming radiation to be quantized. The 'photon' property may then be assigned only to the interaction process whereby the inertia of the electron, upon receipt of radiation, causes it to respond in a quantized manner (Jennison 1973). We suggest that these results may be of interest in the investigation of field models of the electron and other particles.

References

Born M 1957 *Atomic Physics* (Glasgow: Blackie) p 87

Compton A H 1923 *Phys. Rev.* **21** 483–502

Ditchburn R W 1952 *Light* (Glasgow: Blackie) p 327

Jennison R C 1973 *Operational Rigidity* (Canterbury: University of Kent)

Lamb S E and Scully O M 1969 *Polarisation, Matiere et Rayonnement* (Paris: University Press of France) pp 363–9